MATHEMATICS SPECIALISED (MTS315109)

2012 External Examination Information Sheet

TRIGONOMETRY:

$\sin^2 A + \cos^2 A = 1$

$1 + \tan^2 A = \sec^2 A \qquad 1 + \cot^2 A = \operatorname{cosec}^2 A$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$\sin 2A = 2 \sin A \cos A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1 \qquad \cos 2A = 1 - 2\sin^2 A$

$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

$$\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$$

$2 \cos A \sin B = \sin (A + B) - \sin (A - B)$

$$\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$$

$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

$$\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$$

$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$

$$\cos C - \cos D = 2 \sin \frac{C + D}{2} \sin \frac{D - C}{2}$$

CALCULUS:

$$\frac{d \sin^{-1} x}{dx} = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d \cos^{-1} x}{dx} = -\frac{1}{\sqrt{1 - x^2}} \qquad \frac{d \tan^{-1} x}{dx} = \frac{1}{1 + x^2}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c \quad \text{or} \quad -\cos^{-1} \frac{x}{a} + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$\frac{da^x}{dx} = a^x \ln a \qquad \int a^x dx = \frac{a^x}{\ln a} + c$$

$$\frac{d \log_a x}{dx} = \frac{1}{x \ln a} \qquad \int \log_a x dx = \frac{x \ln x - x}{\ln a} + c$$

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx + c$$

Volumes of solids of revolution:

about x-axis $\quad \pi \int_a^b y^2 dx$ $\qquad$ about y-axis $\quad \pi \int_a^b x^2 dy$

SEQUENCES AND SERIES:

Arithmetic Series: $U_n = a + (n-1)d$ (often denoted by ℓ the last term)

$S_n = \frac{n}{2}(2a + (n-1)d)$ or $\frac{n}{2}(a + \ell)$

Geometric Series: $U_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r}$ if $r \neq 1$ or na when $r = 1$

$S_\infty = \frac{a}{1-r}$ if $|r| < 1$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2} \qquad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

The sequence $\{a_n\}$ converges to a finite limit L if, for any $\varepsilon > 0$, $\exists\, N(\varepsilon)$ such that $|a_n - L| < \varepsilon \;\forall\; n > N$.

The sequence $\{a_n\}$ diverges to positive infinity if, for any $\kappa > 0$, $\exists\, N(\kappa)$ such that $a_n > \kappa \;\forall\; n > N$.

The sequence $\{a_n\}$ diverges to negative infinity if, for any $\kappa > 0$, $\exists\, N(\kappa)$ such that $a_n < -\kappa \;\forall\; n > N$.

MacLaurin's series for $f(x)$ is:

$$f(x) = f(0) + f'(0)x + f''(0).\frac{x^2}{2!} + f'''(0).\frac{x^3}{3!} + \ldots + f^{(n)}(0)\frac{x^n}{n!} + \ldots.$$

MATRICES:

Some important transformations are described by the matrices:

Dilation Matrices: $\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}$, Shear Matrices: $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$.

Rotation Matrix: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, Reflection Matrix: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$.

Equation of circle centre (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$

Equation of ellipse centre (h, k) and horizontal semi-axis of length a and vertical semi-axis of length b is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.